MAO-003-001618 Seat No. ________

Third Year B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2018

Mathematics : BSMT-603(A)

(Optimization & Numerical Analysis-II)

Faculty Code : 003

Subject Code : 001618

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate full marks.

1 Answer the following objective type questions briefly in your answer-book : **20**

(1) The linear function in any LPP that is to be optimized is called __________.

(2) What is Hyperplane ?

(3) Define Convex Hull.

(4) What is slack variable ?

(5) Write the Standard form of Linear Programming Problem.

(6) Which method helps in comparing the relative advantage of alternative allocations for all the unoccupied cells simultaneously to solve transportation problem ?

(7) Which three methods are used to obtain an initial solution of transportation problem ?

(8) Allocate in the second row of the following transportation problem by NWCM :

		To Destination		Supply
		W1	W2	
From Origin	F1	2	7	5
	F2	3	3	8
	F3	5	4	3
Demand		7	9	16

(9) What is Assignment Problem ? Explain in very brief.

(10) Which method of Operations Research was developed by a mathematician D. Konig ?

(11) Which difference interpolation formulae are most suitable ?

(12) Write Gauss's forward interpolation formula.

(13) When the concept of divided difference is used ?

(14) Write the relation between divided differences and forward differences.

(15) If $f(x) = x^3 - 2x$ then find $f(2,4,9,10)$.

(16) The formula to find $\int_a^b y dx$ is called __________.

(17) $\int\limits_{x_0}^{x_0+nh} y dx = h[\frac{y_0 + y_n}{2} + y_1 + y_2 + ... + y_{n-1}$ is known as __________ Rule.

(18) Write the formula to find the value of $\int_2^6 \frac{dx}{x}$ by trapezoidal rule.

(19) State Simposon's $\frac{3}{8}$ Rule to find the value of $\int_2^6 \frac{dx}{x}$.

(20) What is the range of p for which the Gauss's backward interpolation formula is useful ?

2 (A) Attempt any **three** : **6**

(1) Define :

(i) Optimum solution of Linear Programming Problem.

(ii) Unbounded solution (w.r.t. Linear Programming Problem).

(2) Find the dual of the following primal linear programming problem :

$$\text{Maximize } z = 3x_1 + 5x_2 + 4x_3 \text{ subject to } 2x_1 + 3x_2 + 0x_3 \le 8,$$

$$0x_1 + 2x_2 + 5x_3 \le 10,\ 3x_1 + 2x_2 + 4x_3 \le 15 \text{ and } x_1, x_2, x_3 \ge 0$$

(3) State the Fundamental Theorem of Linear Programming.

(4) Write full form of LCM and NWCM.

(5) Write General Mathematical form of Assignment Problem.

(6) Define : Optimal solution of transportation problem. Write full form of B.F.S.

(B) Attempt any **Three** : **9**

(1) Obtain the **<u>INITIAL</u>** solution of given transportation problem using **<u>LCM method</u>** :

		TO				Supply
		D_1	D_2	D_3	D_4	
FROM	P_1	2	3	11	7	6
	P_2	1	0	6	1	1
	P_3	5	8	15	9	10
Demand		7	5	3	2	

(2) Obtain the Initial solution of the above transportation problem using VAM.

(3) Obtain the dual of the following :

Minimize

$Z = x_1 + 2x_2$

Subject to

$2x_1 + 4x_2 \leq 160, \quad x_1 = x_2 = 30, \quad x_1 \geq 10$ and $x_1, x_2 \geq 0.$

(4) Explain the steps of the **GRAPHICAL** method to solve the Linear Programming Problems.

(5) Explain the steps of **The Big-M Method**.

(6) Write summary of the general relationship between primal and dual LP Problems.

(C) Attempt any **two** : **10**

(1) Explain steps of **Hungarian method** to solve the **Assignment Problem**.

(2) Find the **OPTIMUM** solution of given **ASSIGNMENT PROBLEM** :

		Job				
		1	2	3	4	5
Men	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

(3) Obtain the **OPTIMUM** solution of given **Transportation Problem** using **MODI** method :

		Destination				
		S_1	S_2	S_3	S_4	Supply
Origin	O_1	21	16	25	13	11
	O_2	17	18	14	23	13
	O_3	32	27'	18	41	19
Demand		6	10	12	15	43

(4) Find **ONLY BFS** and construct **ONLY FIRST TABLE** to solve the following LPP using **SIMPLEX METHOD** (complete solution is not required)

Maximize $Z = 2x_1 + 5x_2 + 7x_3$

subject to the

constraints : $3x_1 + 2x_2 + 4x_3 \le 100,\ x_1 + 4x_2 + 2x_3 \le 100,$

$x_1 + x_2 + x_3 \le 100$ and $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$

(5) Obtain the dual problem of the following primal LP problem :

Minimize $Z_x = 5x_1 + 2x_2 + x_3$

Subject to

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 + 0x_3 \le 3$

$0x_1 + 2x_2 - x_3 \ge 4$

and $x_1, x_2 \ge 0, x_3$ is unrestricted in sign.

3 (A) Attempt any **three** : **6**

(1) Write Lagrange's interpolation formula.

(2) For $x_0 = 1; x_1 = 1$ and $x_2 = 2$ prove that sum of coefficients of Lagrange's interpolation is 1.

(3) If $f(1) = 2, f(1,3) = 13. f(1,3,6) = 10$ and

$f(1,3,6,9) = 1$ then prove that $f(x) = x^3 + 1$.

(4) Prove that the value of any divided difference is independent of the order of the arguments.

(5) In usual notation prove that

$$D = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + ...\right].$$

(6) Derive relation between divided differences and forward differences.

(B) Attempt any **three** : **9**

(1) If $f(x) = x^3$ then find $f(1,3,5,7)$.

(2) Derive Simpson's 1/3 Rule.

(3) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by Simpson's $\frac{3}{8}$ Rule.

(4) Solve $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ by Taylor's series method.

(5) Explain Picard's method to solve the differential equation $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

(6) Find the values of y at $x = 0.2, 0.4, 0.8, 1$ by Euler's method $\frac{dy}{dx} = 2x + y,\quad y(0) = 1$.

(C) Attempt any **two** : **10**

(1) Obtain the value of $f'(90)$ using Stirling's formula to the following data :

x	60	75	90	105	120
$f(x)$	28.2	38.2	43.2	40.9	37.7

(2) Derive Bessel's formula for central differences.

(3) Derive Simpson's $3/8$ rule.

(4) Explain Range's Method to solve differential equation.

(5) Find a polynomial satisfied by the following table :

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335